# Examen analyse 2



## exo 1 :

(a) on sait que $\lim\limits_{n\to+\infty} \left(-\frac{1}{5}\right)^n = 0$ car $\left|-\frac{1}{5}\right| < 1$

et $\lim\limits_{n\to+\infty} \dfrac{2n^2+1}{n^2+n+3} = 2$ car $\dfrac{2n^2+1}{n^2+n+3} = \dfrac{2n^2\,(1+\overbrace{1/2n^2}^{\to 0})}{n^2\,(1+\underbrace{1/n}_{\to 0}+\underbrace{3/n^2}_{\to 0})}$

donc $\lim\limits_{n\to+\infty} u_n = 2$

(b) on a : $\dfrac{v_{n+1}}{v_n} = \dfrac{\sqrt{n+1}}{2^{n+1}} \times \dfrac{2^n}{\sqrt{n}} = \sqrt{1+\frac{1}{n}} \times \dfrac{1}{2} \xrightarrow[n\to+\infty]{} \dfrac{1}{2} < 1$

donc par le critère de d'Alembert la série de terme général $v_n$ est convergente.

## exo 2 :

(a)

$f(x) = e^{2x}$ $\qquad f(0) = 1$

$f'(x) = 2e^{2x}$ $\qquad f'(0) = 2$

$f''(x) = 4e^{2x}$ $\qquad f''(0) = 4$

$f^{(3)}(x) = 8e^{2x}$ $\qquad f^{(3)}(0) = 8$

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2} f''(0) + \frac{x^3}{6} f^{(3)}(0) + x^3 \varepsilon(x) \text{ avec } \varepsilon(x) \xrightarrow[x\to 0]{} 0$$

$$= 1 + 2x + 2x^2 + \frac{4}{3}x^3 + x^3\varepsilon(x)$$

(b)

$g(x) = \sqrt{1+x}$ $\qquad g(0) = 1$

$g'(x) = \dfrac{1}{2\sqrt{1+x}}$ $\qquad g'(0) = \dfrac{1}{2}$

$g''(x) = -\dfrac{1}{4(1+x)\sqrt{1+x}}$ $\qquad g''(0) = -\dfrac{1}{4}$

$$g(x) = g(0) + x g'(0) + \frac{x^2}{2} g''(0) + x^2 \varepsilon(x) \text{ avec } \varepsilon(x) \xrightarrow[x\to 0]{} 0$$

$$= 1 + \frac{x}{2} - \frac{x^2}{8} + x^2 \varepsilon(x)$$

exercice 3

(a) $$\int_0^{\pi/2} x \cos x \, dx = \int_0^{\pi/2} f(x) g'(x) dx = \left[ f(x) g(x) \right]_0^{\pi/2} - \int_0^{\pi/2} f'(x) g(x) dx$$

$f(x) = x \qquad f'(x) = 1$

$g'(x) = \cos x \qquad g(x) = \sin x$

$$\int_0^{\pi/2} x \cos x \, dx = \left[ x \sin x \right]_0^{\pi/2} - \int_0^{\pi/2} \sin x \, dx$$

$$= \frac{\pi}{2} - \left[ -\cos x \right]_0^{\pi/2}$$

$$= \frac{\pi}{2} - 1$$

(b) $$\int_0^1 \frac{x}{1+x^2} dx = \frac{1}{2} \int_0^1 \frac{2x}{1+x^2} dx = \frac{1}{2} \int_0^1 \frac{u'(x) \, dx}{1+u(x)}$$

avec $u(x) = x^2$

$u'(x) = 2x$

$$\int_0^1 \frac{x}{1+x^2} dx = \frac{1}{2} \int_0^1 \frac{du}{1+u} = \frac{1}{2} \left[ \ln(1+u) \right]_0^1$$

$$= \frac{\ln 2}{2}$$

exercice 4 :

$f(x,y) = x^2 e^{-y}$

$\frac{\partial f}{\partial x}(x,y) = 2x e^{-y} \qquad \frac{\partial f}{\partial y}(x,y) = -x^2 e^{-y}$

$\frac{\partial^2 f}{\partial x^2}(x,y) = 2 e^{-y} \qquad \frac{\partial^2 f}{\partial y^2}(x,y) = x^2 e^{-y}$

$\frac{\partial^2 f}{\partial x \partial y}(x,y) = \frac{\partial^2 f}{\partial y \partial x}(x,y) = -2x e^{-y}$ (par le thm de Schwartz) exercice 5 :

$f(x,y) = (x-1)^2 + (y+1)^2$

(a) $\frac{\partial f}{\partial x}(x,y) = 2(x-1)$

$\frac{\partial f}{\partial y}(x,y) = 2(y+1)$

(b) $\frac{\partial f}{\partial x}(x_0,y_0) = 0 \iff 2(x_0-1) = 0 \iff x_0 = 1$

$\frac{\partial f}{\partial y}(x_0,y_0) = 0 \iff 2(y_0+1) = 0 \iff y_0 = -1$

Donc $f$ admet un seul point critique en $(x_0,y_0) = (1,-1)$

(c) On a : $f(x_0,y_0) = (1-1)^2 + (-1+1)^2 = 0$

et $f(x,y) \geq 0 \quad \forall (x,y) \in \mathbb{R}^2$

$f(x,y) > 0$ pour $(x,y) \neq (x_0,y_0)$

donc $(x_0,y_0)$ est un minimum global.

C'est le seul extremum de la fonction $f$.